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Direct Cerenkov excitation of waveguide modes by a mobile ionospheric heater

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Abstract. A comprehensive theoretical analysis of direct Cerenkov excitation of the Earth ionosphere waveguide using ionospheric heating is presented. The model relies on transient ionospheric heating with a heater spot moving horizontally at the bottom of the waveguide with speed close to the speed of light. The cases of isotropic ionospheric conductivity, corresponding to heating altitudes below 70 km, and of anisotropic conductivity, corresponding to higher heating altitudes, are examined separately. It is found that enhanced radiation coupling requires that the speed of the heater approaches the speed of light. For the anisotropic case, such enhancement occurs independently of the direction of motion, while for the isotropic case, motion parallel to the ambient electric field is required.

1. Introduction

The controlled generation of coherent low-frequency waves in the ULF/ELF/VLF range using amplitude-modulated ionospheric heating with an HF transmitter in the auroral zones and midlatitudes has been investigated extensively experimentally and theoretically [*Getmantsev et al.*, 1974; *Stubbe et al.*, 1981; *Barr and Stubbe*, 1984; *Chang et al.*, 1981; *Beljaev et al.*, 1987; *Papadopoulos et al.*, 1989]. The basic physics of the generation is as follows. Absorption of the HF energy in the lower ionosphere modifies locally the ambient conductivity, leading to the generation of a polarization electric field and current inside the modified region. The surrounding magnetized plasma responds to the temporal modification by driving two field-aligned currents carried by helicon or whistler waves [*Papadopoulos et al.*, 1994a], which close across the magnetic field by a Hall current. A current loop forms, which expands upward for the duration of the HF pulse. The loop damps quickly following the termination of the HF pulse. As a result, amplitude-modulated heating generates an equivalent horizontal magnetic moment M in the lower ionosphere given by

$$M = E_a \Delta\Sigma A \exp(i\omega_0 t) \tag{1}$$

where E_a is the ambient electric field, $\Delta\Sigma$ the value of the modified conductance, A the area of the loop, and ω_0 the frequency of the HF amplitude modulation. The power radiated by the oscillatory moment M couples to the waveguide formed by the conducting ground and the ionosphere and propagates over very large distances with small attenuation [*Kotik et al.*, 1978; *Papadopoulos et al.*, 1990a]. It should be noted here that a current loop with magnetic moment in the opposite direction forms in the downward direction. However, the area of the loop is much smaller than A because of the larger collision frequency at the lower height, and its effect on the radiation can be neglected [*Papadopoulos et al.*, 1990b, 1994a].

The objective of this paper is to present a comprehensive analytical model of an alternative concept leading to injection of low-frequency power in the Earth ionosphere waveguide (EIW) using controlled ionospheric heating. The concept is based on direct excitation of the waveguide by moving the ionospheric heater in the horizontal direction with a speed that matches the phase velocity of the desired waveguide mode. Its mathematical description follows the well-known Cerenkov emission [*Landau and Livshitz*, 1960], which occurs when a charge moves in a medium with speed exceeding the phase speed of the waves the medium supports. The wave emission is due to the polarization electric field induced in the

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medium by the motion of the charge. As mentioned above, ionospheric heating in regions penetrated by currents or electric fields induces polarization fields and currents. One would therefore expect that polarization fields induced by a heater beam moving horizontally with speeds matching or exceeding the phase speed of the waveguide modes will couple energy to the EIW by a process analogous to Cerenkov emission. The possibility for such an effect in the VLF range was previously noted by *Kotik et al.* [1986] and *Borisov et al.* [1991]. In fact, *Kotik et al.* [1986] performed an experiment in which they used a heater as an interferometer with a baseline d and frequency difference of the heating source Δf. By adjusting the values of d and Δf they created a supraluminal motion of the heated region. They noted that there was significant directional gain for frequencies between 8 and 10 kHz over the power produced by HF amplitude modulation in the absence of motion.

We present below a comprehensive theoretical analysis of Cerenkov excitation of the EIW by a moving ionospheric source. Emphasis is placed on the elucidation of the regimes of operation, the requirements for the validity of the theory, and the expected observables. The aim of the study is to provide guidelines for the conduct of proof of principle experiments. This is particularly opportune since the availability of the plasma HF Active Auroral Research Program (HAARP) heater at the end of 1996, will provide an excellent facility for the conduct of such experiments [*Papadopoulos et al.*, 1995]. The paper is organized as follows. Section 2 presents a brief review of the fundamentals of HF heating of the lower ionosphere as a function of the heater parameters and the local ionospheric properties. The form of the polarization current generated by a moving heater is also described. Section 3 presents an analysis of the low-frequency power coupled in the EIW by the moving polarization current. The EIW is modeled by a simple two-layer model with a sharp boundary between the vacuum and the plasma. The heating is assumed to occur at the boundary. Two cases are examined separately. In the first the layer boundary is taken below 70 km, where $\Omega_e \ll \nu_e$, resulting in isotropic conductivity. In the second the layer boundary is taken above 70 km, where $\Omega_e \gg \nu_e$, resulting in anisotropic conductivity. Section 4 examines the region of validity of the theory and its impact on the design of experiments. The last section summarizes the results and discusses some practical applications of the concept.

2. Ionospheric Heating Considerations

The auroral electrojet current system is driven by the solar wind flow past the geomagnetic field. The currents flow along the magnetic field and close by cross-field currents at altitudes between 60 and 110 km. For an ambient electric field $\mathbf{E}_a$ imposed by the solar wind across the magnetic field $\mathbf{B}_0$ there are two dominant currents, the Hall and Pedersen, given by

$$\mathbf{J}_H = -\frac{1}{4}\frac{\omega_e^2}{\Omega_e^2}\frac{\Omega_e}{1+\nu_e^2/\Omega_e^2}\frac{\mathbf{E}_a}{|\mathbf{B}_0|}\times\mathbf{B}_0 \equiv \boldsymbol{\sigma}_H\cdot\mathbf{E}_a \qquad (2)$$

$$\mathbf{J}_P = -\frac{1}{4\pi}\frac{\omega_e^2}{\Omega_e^2}\frac{\nu_e}{1+\nu_e^2/\Omega_e^2}\mathbf{E}_a \equiv \sigma_P\mathbf{E}_a \qquad (3)$$

In (2) and (3), ω_e, Ω_e, and ν_e are the electron plasma, cyclotron, and electron-neutral collision frequencies. The collision frequency ν_e is a function of the electron temperature T_e, which, in the parameter region of interest can be approximated by

$$\nu_e(T_e) = \nu_{e0}\frac{T_e}{T_0} \qquad (4)$$

where ν_{e0} and T_0 are the ambient values. Under the action of HF radio waves with rms amplitude E_0 and frequency ω_0 the local electron temperature is given by [*Gurevich*, 1978]

$$\frac{\partial T_e}{\partial t} = \frac{1}{3}\frac{e^2E_0^2\nu_e}{m(\omega_0^2+\nu_e^2)} - \nu_e\delta(T_e - T_0) \qquad (5)$$

where δ is the average energy lost in one collision between electrons and neutrals. For strong heating with $T_e - T_0 > T_0$ and assuming $\omega_0 \gg \nu_e$ and Ω_e, the value of ν_e as a function of time following a sudden switch-on of the HF power at $t = 0$ is given by

$$\nu_e/\nu_{e0} = \frac{1 + E_0^2/E_p^2}{1 + E^2/E_p^2\exp\left[-\delta\nu_{e0}t(1 + E_0^2/E_p^2)\right]} \qquad (6)$$

where E_p is the plasma field defined by [*Gurevich*, 1978]

$$E_p = \left(\frac{3T_0\delta m\omega_0^2}{e^2}\right)^{1/2} \qquad (7)$$

From (6) it follows that ν_e reaches a stationary value $\nu_e/\nu_0 = 1 + E_0^2/E_p^2$ on a timescale $\tau \approx [\delta\nu_{e0}(1 + E_0^2/E_p^2)]^{-1}$. Notice that this time is shorter for $E_0/E_p > 1$ and approaches $1/\delta\nu_{e0}$ for $E_0/E_p < 1$. Following the end of the HF pulse, the temperature returns to its ambient value on a time $1/\delta\nu_{e0}$.

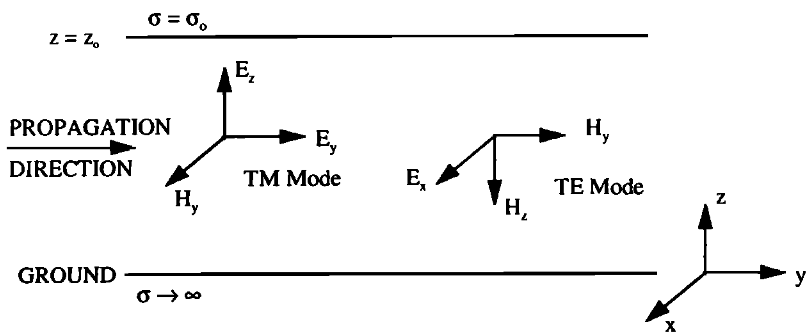

Figure 1a. Structure of transverse magnetic (TM) and transverse electric (TE) modes for the Earth ionosphere waveguide.

We consider next an HF pulse whose energy is absorbed at an altitude z_0 in the lower ionosphere over a region taken as a Gaussian with radial extent a and vertical extent L_z. If the HF beam remains on over a time $t \geq \tau$, the modification in the collision frequency will induce currents given by

$$\frac{\Delta J_H(\mathbf{r})}{|J_H|} = \left(1 - \frac{1 + \nu_{e0}^2/\Omega_e^2}{1 + (\nu_{e0}^2/\Omega_e^2)g^2}\right) S(\mathbf{r}) \tag{8a}$$

$$\frac{\Delta J_p(\mathbf{r})}{|J_p|} = \left(1 - g\,\frac{1 + \nu_{e0}^2/\Omega_e^2}{1 + (\nu_{e0}^2/\Omega_e^2)g^2}\right) S(\mathbf{r}) \tag{8b}$$

$$g \equiv 1 + \frac{E_p^2}{E_0^2} \quad S(\mathbf{r}) = \exp\left(-\frac{r^2}{a^2}\right) \exp\left[-\frac{(z-z_0)^2}{L_z^2}\right] \tag{8c}$$

Consider next the case that the heater moves with velocity $\mathbf{u}$. In this case the modified current will again be given by (8) but with $S(\mathbf{r})$ replaced with $S(\mathbf{r} - \mathbf{u}t)$.

These currents form the external source in the wave equation which excites the waveguide. We define it thus as $\mathbf{J}_{\mathrm{ext}}(\mathbf{r}, t)$

$$\mathbf{J}_{\mathrm{ext}}(\mathbf{r}, t) = \Delta\mathbf{J}_H(\mathbf{r} - \mathbf{u}t) + \Delta\mathbf{J}_p(\mathbf{r} - \mathbf{u}t). \tag{9}$$

3. Cerenkov Emission in an Isotropic Ionosphere

The physics of the interaction is best illustrated by considering the case of isotropic conductivity. In practice this is the case for heating altitude $z_0 \ll 70$ km, corresponding to $\nu_e \gg \Omega_e$. We furthermore consider a simplified model with the ground as an infinite conducting layer, followed by a vacuum up to $z = z_0$ (Figure 1). Above z_0 the dielectric constant is taken as homogeneous with

$$\varepsilon = 1 - \frac{4\pi i \sigma_0}{\omega} = 1 - i\,\frac{\omega_e^2}{\omega \nu_e} \tag{10}$$

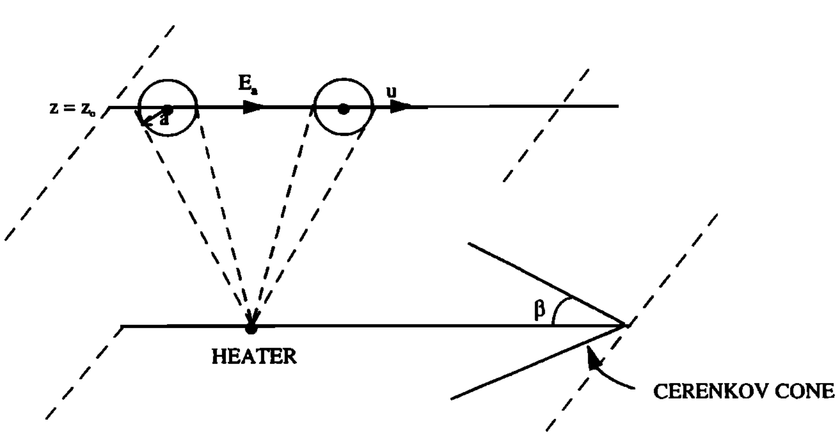

Figure 1b. Schematic of Cerenkov generation of ELF pulses by sweeping of the HF heater at a speed larger than the phase speed of the TM mode.

where ω_e is the plasma frequency. Since $\nu_e \gg \Omega_e$

$$\mathbf{J}_{\text{ext}}(\mathbf{r}, t) = \mathbf{J}_p(1 - 1/g)S(\mathbf{r} - \mathbf{u}t). \tag{11}$$

For $\mathbf{E}_a = E_a\hat{e}_y$ and $\mathbf{u} = \mathbf{u}\hat{e}_y$ the wave equation for the transverse electric (TE) and transverse magnetic (TM) modes can be written as

$$\frac{d}{dz}\frac{k_0^2\varepsilon}{k_0^2\varepsilon - k_\perp^2}\frac{d}{dz}\psi_1 + k_0^2\varepsilon\psi_1 = \frac{4\pi i\omega}{c^2}k_yJ_0(z, k_\perp)\delta(\omega - k_yu) \tag{12a}$$

$$\left(\frac{d^2}{dz^2} - k_\perp^2\right)\psi_2 + k_0^2\varepsilon\psi_2 = \frac{4\pi i\omega}{c^2}k_xJ_0(z, k_\perp)\delta(\omega - k_yu) \tag{12b}$$

$$J_0(z, k_\perp) \equiv E_a(\Delta\Sigma)\frac{a^2}{4\pi}\exp\left(-\frac{a^2k_\perp^2}{4}\right)\delta(z - z_0) \tag{13a}$$

$$\Delta\Sigma = \sqrt{\pi}\Delta\sigma L_z \tag{13b}$$

where k_0 is the free space wave number. Consider first the generation of TM waves. We look for solutions with

$$\psi_1 = b_1 \sin\sqrt{k_0^2 - k_\perp^2}z \qquad 0 < z < z_0 \tag{14a}$$

$$\psi_2 = b_2 \sin\sqrt{k_0^2\varepsilon - k_\perp^2}(z - z_0) \qquad z \geq z_0 \tag{14b}$$

From (12) and (14) the vertical component of the electric field on the ground $E_z(z = 0, y) \equiv E_z(0)$ is given by

$$E_z(z = 0) = ia^2E_a \int\int\int \frac{d\omega\, dk_x\, dk_y}{\varepsilon(\omega) - \omega} \cdot \frac{\sqrt{k_0^2\varepsilon - k_\perp^2}\Delta\Sigma k_y\delta(\omega - k_yV)e^{i[\omega t-(k_yy+k_xx)]}}{\sqrt{k_0^2 - k_\perp^2}\sin\sqrt{k_0^2 - k_\perp^2}z_0 - i\dfrac{\sqrt{k_0^2\varepsilon - k_\perp^2}}{\varepsilon}\cos\sqrt{k_0^2 - k_\perp^2}z_0} \cdot \exp\left(-\frac{a^2k_\perp^2}{4}\right) \tag{15}$$

The zeros of the denominator in (15) correspond to the different modes of the waveguide. It is well known that in the frequency range we are interested in, up to several kilohertz, the main contribution to the field in the wave zone is due to the TM_0 mode. For $\sigma/\omega \gg 1$ the eigenvalue of this mode is determined by the equation

$$(k_0^2 - k_\perp^2)z_0 + \frac{\omega}{4\pi\sigma}\sqrt{k_0^2\varepsilon - k_\perp^2} = 0 \tag{16}$$

After integration in (15) with respect to k_z and neglecting small terms we obtain for the radiating field

$$E_z(z = 0) = -\frac{i\pi^{1/2}E_av^{1/2}}{\sigma^{1/2}z_0\beta c}a^2\int dk_y \cdot e^{ik_y(y\pm\beta x-vt)}\sqrt{-ik_y}\Delta\Sigma\exp\left(-\frac{a^2k_\perp^2}{4}\right) \tag{17}$$

where $\beta = (v^2/c^2 - 1)^{1/2}$ and the signs $\pm$ before βx correspond to the regions $x > 0$ and $x < 0$. The meaning $\sqrt{-ik_y}$ is chosen in the same way as mentioned earlier (see (14b)). Assuming that the perturbation of the conductivity is in the horizontal plane, we obtain for the radiating field

$$E_z(z = 0) \approx -\frac{\sqrt{2\pi a}\Delta\Sigma_0v^{1/2}}{z_0\sigma^{1/2}c\beta}E_a\left[1.7\phi\left(\frac{3}{4}; \frac{1}{2}; \frac{\xi^2}{a^2}\right) - 2.6\frac{\xi}{a}\phi\left(-\frac{3}{4}; \frac{3}{2}; \frac{\xi^2}{a^2}\right)\right] \tag{18}$$

Here $\xi = vt - y \mp \beta x$, the sign $\mp$ before βx corresponds to the regions $x > 0$ and $x < 0$, and $\phi(\alpha, \beta, x)$ is the confluent hypergeometric function. According to (18), the radiation propagates in the horizontal plane as two temporally localized wave packets making angles $\alpha = \pm\arctan\beta$ to the y axis. In the limit $v \gg c$ the field amplitude diminishes as a power law with increasing v. On the contrary, radiation grows as $\beta \to 0$, that is, if the speed v approaches the speed of light. In this case it is necessary to retain the small imaginary terms in (16). The result is valid up to $\beta = (ca/\sigma z_0^2)^{1/4}$.

We consider next the case when the electric field $\mathbf{E}_a = \hat{e}_xE_a$ is perpendicular to the direction of motion. In this case we obtain the field in the wave zone $|x| \gg a$ as

$$E_z(z = 0) \approx -\frac{\sqrt{2\pi a}\Delta\Sigma v^{1/2}}{z_0\sigma^{1/2}c}E_a \cdot \left[1, 7\phi\left(\frac{3}{4}; \frac{1}{2} - \frac{\xi^2}{a^2}\right) - 2.6\xi\phi\left(-\frac{3}{4}; \frac{3}{2}; \frac{\xi^2}{a^2}\right)\right] \tag{19}$$

The main difference from the previous case is that E_z does not depend on β. As a result, the field enhancement for $v \to c$ is absent. This implies that for isotropic media, only the motion along $\mathbf{E}_a$ leads to enhanced radiation amplitude. According to (18) and (19), the spectral amplitude diminishes for frequencies $\omega > 2c/a$. For $a \approx 20$ km and $v \approx c$ the characteristic frequency is 5 kHz, and the main mode excited in the waveguide is the TM_0 mode. Later on we consider only the contribution of this mode to the radiating field in the wave zone.

4. Cerenkov Radiation in Anisotropic Media

The anisotropy of the medium due to the external magnetic field makes the problem of ELF generation more complicated. For anisotropic media it is necessary to take into account the interaction of the waves with different polarization. In this case the external electric field $\mathbf{E}_a$ generates currents both in the direction and across the direction of the electric field. We write the external source as

$$\mathbf{J}_{\text{ex}} = \{\Delta\sigma_p E_a, \Delta\sigma_H E_a\} \tag{20}$$

and proceed to investigate the solution of the wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial}{\partial t}\boldsymbol{\sigma}\cdot\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\mathbf{J}_{\text{ex}} \tag{21}$$

The vacuum solution for the electric field is given again by (12a). To solve (21) for an anisotropic medium, it is convenient to introduce instead of the components E_x and E_y their linear combinations corresponding to the ordinary and extraordinary modes,

$$\phi_1 = E_x + iE_y \qquad \phi_2 = E_x - i\tilde{E}_y \tag{22}$$

where the tilde denotes temporal transform. If we assume that the longitudinal component of the conductivity in the ionosphere $\sigma_{\parallel} = \omega_{pe}^2/4\pi\nu_e$ is infinite, we obtain for $\phi_{1,2}$

$$\frac{d^2\phi_1}{dz^2} + \left(k_0^2 - k_\perp^2 - \frac{4\pi i\omega\sigma_p}{c^2} + \frac{4\pi i\omega\sigma_H}{c^2}\right)\phi_1 + \tfrac{1}{2}(k_x + ik_y)\phi_2 = \frac{4\pi i\omega}{c^2}E_a(\Delta\sigma_p - i\Delta\sigma_H) \tag{23}$$

$$\frac{d^2\phi_1}{dz^2} + \left(k_0^2 - k_\perp^2 - \frac{4\pi i\omega\sigma_p}{c^2} + \frac{4\pi i\omega\sigma_H}{c^2}\right)\phi_2 + \tfrac{1}{2}(k_x + ik_y)\phi_1 = \frac{4\pi i\omega}{c^2}E_a(\Delta\sigma_p + i\Delta\sigma_H) \tag{24}$$

In the general case the system of (23) and (24) for $\sigma_p = \sigma_p(z)$ and $\sigma_H = \sigma_H(z)$ can be solved only numerically. To obtain an approximate analytical solution, we assume the following.

1. The Hall conductivity outside the waveguide is large enough so that for the relevant frequencies ω and scales k there is a small parameter

$$c^2|k_x \pm ik_y|^2/4\pi\omega\sigma_H \ll 1 \tag{25}$$

2. The thickness of the region where the source is located Lz is relatively thin

$$(4\pi\omega\sigma_H/c^2)^{1/2}L_z < 1 \tag{26}$$

3. Outside the waveguide the conductivities σ_H and σ_p change smoothly with height

$$\frac{4\pi\omega\sigma_H h_H^2}{c^2} > 1 \qquad \frac{4\pi\omega\sigma_p h_p^2}{c^2} > 1 \tag{27}$$

where

$$h_H = \left|\frac{d\ln\sigma_H}{dz}\right|^{-1} \qquad h_p = \left|\frac{d\ln\sigma_p}{dz}\right|^{-1}.$$

We discuss the correctness of the above conditions for the frequency range of interest ($f \sim 1$–5 kHz). In the anisotropic ionosphere the formation of the upper boundary of the waveguide can be assumed as the height $z = z_0$, where the Hall conductivity becomes sufficiently large, $\sigma_H > \omega/4\pi$. For daytime conditions it corresponds to the heights $z_0 \approx 65$–70 km (depending on the frequency ω and parameters of the media). At such heights the Pedersen conductivity σ_p is considerably smaller than the Hall conductivity σ_H. The ordinary wave according to (24) is reflected to the Earth, while the extraordinary wave penetrates the ionosphere. For frequencies $f \sim 1$–5 kHz and scales $a \sim 10$–30 km at heights $z > z_0$, condition 1 is fulfilled. This, to first order, allows us to split the equation for the ordinary and the extraordinary modes.

The second condition demands that the effective thickness L_z, where the source is localized, be smaller than the thickness of the skin depth. Otherwise the thickness of the source is limited to the skin depth. This condition allow us to use the approximation of source localized at a fixed height

$$J_{\text{ext}}(\omega, k, z) = \{\Delta\Sigma_p, -\Delta\Sigma_H\}E_0\delta(\omega - k_y v)\delta(z - z_0) \tag{28}$$

where $\Delta\Sigma_{p,H} = \int \Delta\sigma_{p,H}(z')\, dz'$. It should be mentioned that for frequencies of the order of several kilohertz the effective current becomes weaker than (28) because of screening outside the waveguide. The third condition allows us to use the approximation of geometric optics in computing the radiation outside the waveguide.

We choose for $\omega > 0$ as one of the solutions of the function ϕ_1, which describes the wave propagating upward, and as the second solution of the function ϕ_2, which corresponds to the attenuated wave. As a result,

$$\phi_1 = \frac{b_1}{\sqrt{q_1}} \exp\left(-i\int_{z_0}^{z} q_1\, dz\right)$$
$$\phi_2 = \frac{b_2}{\sqrt{q_2}} \exp\left(-i\int_{z_0}^{z} q_2\, dz\right) \tag{29}$$

where

$$q_1 = \sqrt{(4\pi\omega/c^2)(\sigma_H - i\sigma_p)},$$
$$q_2 = \sqrt{(4\pi\omega/c^2)(\sigma_H + i\sigma_p)},$$

and b_1 and b_2 are arbitrary constants. Taking for $\omega < 0$ the meaning $\sqrt{\omega} = -i\sqrt{|\omega|}$, we find that if $\omega < 0$, the function ϕ_1 corresponds to the attenuated wave and ϕ_2 corresponds to the propagating upward wave.

We next consider the boundary conditions for the horizontal component at $z = z_0$. For this purpose we use equations similar to (12) in which we take into account the tensor character of the dielectric permeability $\boldsymbol{\varepsilon}$

$$\frac{d}{dz}\frac{k_0^2\varepsilon_\parallel}{k_0^2\varepsilon_\parallel - k_\perp^2}\frac{d\psi_1}{dz} + k_0^2\varepsilon_\perp\psi_1 - k_0^2\varepsilon_{12}\psi_2$$
$$= \frac{4\pi i\omega}{c^2} E_a(k_x\Delta\sigma_p - k_y\Delta\sigma_p)\delta(\omega - k_y V)$$
$$\left(\frac{d^2}{dz^2} - k_\perp^2\right)\psi_2^2 + k_0^2\varepsilon_\perp\psi_2 - k_0^2\varepsilon_{12}\psi_1 \tag{30}$$
$$= \frac{4\pi i\omega}{c^2} E_a(k_x\Delta\sigma_p + k_y\Delta\sigma_p)\delta(\omega - k_y V)$$

Here $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the diagonal and $\varepsilon_{12} = -\varepsilon_{21}$ the off-diagonal components of the tensor $\boldsymbol{\varepsilon}$. For the selected model the conductivities change abruptly at the upper boundary. We obtain with the help of (30) the boundary conditions in the form

$$\left.\frac{d\psi_1}{dz}\right|_{z_0+0} - \frac{k_0^2}{k_0^2 - k_\perp^2}\left.\frac{d\psi_1}{dz}\right|_{z_0+0}$$
$$= \frac{4\pi i\omega}{c^2} E_a(k_x\Delta E_p - k_y\Delta E_H)\delta(\omega - k_y v)$$
$$\left.\frac{d\psi_2}{dz}\right|_{z_0+0} - \left.\frac{d\psi_2}{dz}\right|_{z_0+0} \tag{31}$$
$$= \frac{4\pi i\omega}{c^2} E_a(k_x\Delta E_p + k_y\Delta E_H)\delta(\omega - k_y v)$$

At the same time the horizontal components E_x and E_y are continuous at the boundary $z = z_0$. Substituting in (31) the solution (29) for the field outside the waveguide and eliminating constants b_1 and b_2, it is easy to obtain for the vertical component of the electric field the approximate expression

$$E_x(z=0) = -\frac{E_a}{z_0^2\sqrt{\sigma_p^2 + \sigma_H^2}}\int\int\int d\omega\, dk_z\, dk_y$$
$$\cdot\, \delta(\omega - k_y v)e^{i\omega t - i(k_x x + k_y y)}$$
$$\cdot\, \frac{z_0(\Delta\Sigma_p k_y + \Delta\Sigma_H k_x)(iq_1 - q_2) - i2(k_x\Delta\Sigma_p + k_y\Delta\Sigma_H)}{k_0^2 - k_\perp^2}$$
$$- \frac{iz_0(\Delta\Sigma_p k_x + \Delta\Sigma_H k_y)(iq_1 + q_2)}{k_0^2 - k_\perp^2} \tag{32}$$

After integrating (32) with respect to ω and wave vectors $\mathbf{k}$ we obtain the expression for E_z in the wave zone. For $x > 0$ it takes the form

$$E_z(0) = \frac{\sqrt{\pi}(1+i)a^2E_a}{\beta z_0^2\sqrt{\sigma_p^2 + \sigma_H^2}cv}\left(\frac{4v^2}{a^2(1+\beta^2)}\right)^{3/4}$$
$$\cdot\, (\Delta\Sigma_p^0 + \beta\Delta\Sigma_H^0)z_0(i\sqrt{\sigma_H - i\sigma_p} - \sqrt{\sigma_H + i\sigma_p})$$
$$- i(\beta\Delta\Sigma_p^0 - \Delta\Sigma_H^0)z_0(i\sqrt{\sigma_H - i\sigma_p} + \sqrt{\sigma_h + i\sigma_p})$$
$$\cdot \left[0.6\phi\left(\frac{3}{4}; \frac{1}{2}; -\frac{(\Delta t)^2v^2}{(1+\beta^2)a^2}\right)\right.$$
$$\left. - 0.9\frac{v\Delta t}{a(1+\beta^2)^{1/2}}\phi\left(\frac{9}{4}; \frac{1}{2}; -\frac{(\Delta t)^2v^2}{a(1+\beta^2)^2}\right)\right]$$

$$- i2(\beta\Delta\Sigma_p^0 + \Delta\Sigma_H^0)\frac{cv}{a\sqrt{1+\beta^2}}e^{(\Delta t)^2v^2/(1+\beta^2)c^2} \quad (33)$$

Here $\Delta t = t(y + \beta v/v)$ and $\Delta\Sigma_{p,H}$ are the changes in the integrated Pedersen and Hall conductivities in the center of the heated spot.

It follows from (33) that the emission becomes more effective when β approaches zero, that is, if the speed of the heated spot approaches the speed of wavelight. On the contrary, for large β the emission strength is reduced. If the ionospheric conductivity is sufficiently large

$$\sigma_H z_0^2 \gg ca \quad (34)$$

we obtain the approximate expression for the ELF field when $\beta \ll 1$ as

$$E_z(0) = \frac{2\sqrt{2\pi a}}{\beta z_0 \sigma_H^{1/2} c^{1/2}} E_0(\Delta\Sigma_p^0 + \beta\Delta\Sigma_H^0)$$

$$\cdot\left[0.6\phi\left(\frac{3}{4};\frac{1}{2};\frac{\Delta t^2 c^{2/a^2}}{a^2}\right) - 0.9\frac{c\Delta t}{a}\phi\left(\frac{9}{4};\frac{3}{2};\frac{\Delta t^2 c^2}{a^2}\right)\right] \quad (35)$$

The result (35) is valid if $\sigma_H > \sigma_p$ and σ is not too small

$$1 > \beta \gtrsim \beta_0 = [ca/(z_0^2\sigma_H)]^{1/2} \quad (36)$$

For smaller β it is necessary to retain dissipative terms in the denominator in (32) that influence the value of the residue. Estimates show that the maximum of radiation corresponds to $\beta \sim \beta_0$. Comparing (35) with the result obtained in the previous section, we find that in both cases similar intensification of the radiation exists for $\beta \to 0$. However, in the case of isotropic media, such an effect is absent if the vector of the external field $\mathbf{E}_a$ is orthogonal to the direction of the motion.

5. Concluding Remarks

We presented above an analysis of the excitation of the EIW by using an ionospheric heating source moving at the bottom of the waveguide with speed close to the speed of light. It should be noted that contrary to the amplitude modulation in which a monochromatic wave at the modulated frequency is generated the Cerenkov emission generates broadband radiation, whose dominant frequency is controlled by the transit time of the heater over its horizontal size. The overall spectrum $E_z(\omega)$ is given by

$$E_z(\omega) = Ae^{i(\omega/c)(y+\beta x)}\left(\frac{a\omega}{2c}\right)^{1/2} e^{-a^2\omega^2/4c^2}\phi\left(0;\frac{5}{4};\frac{a^2\omega^2}{4c^2}\right)$$

$$- i0.4\frac{c}{a}\frac{\partial}{\partial\omega}\left[\left(\frac{a\omega}{2c}\right)^{7/2} e^{-a^2\omega^2/4c^2}\phi\left(1;\frac{11}{4};\frac{a^2\omega^2}{4c^2}\right)\right] \quad (37)$$

where

$$A = 0.6\left(\frac{2\pi a}{\sigma_H c}\right)^{1/2}\frac{\Delta\Sigma_p^0 + \Delta\Sigma_H^0}{\beta_H z_0}E_a$$

and $\phi(\alpha, \beta, z)$ is a confluent hypergeometric function. It follows from (37) that the intensity of the radiation quickly diminishes at large frequencies ($\omega > 2c/a$). The maximum of the spectrum is at the frequency range

$$f \sim f^* = c/2\pi a \quad (38)$$

At the same time, spectral amplitude, according to (37), changes slowly for small frequencies. It is interesting to find out how the spectral amplitude at $f = f^*$ depends on the radius a. Let us assume that the full power of the heater is constant. The increase of the heating of electrons falls $\sim a^{-2}$, hence the collision frequency of electrons also becomes smaller. So the increase of radius a in the case of weak heating with $\nu_e \sim T_e$ leads to considerable decrease in the amplitude of the spectral maximum.

In our analysis it was implicitly assumed that the source moves uniformly along the y axis between $y = -\infty$ and $y = +\infty$. It is important to determine the effect on the excitation amplitude resulting from a finite size sweep with length Δy. The analysis of the appendix shows that the requirement for optimum Cerenkov emission is that the sweeping length L satisfy the inequality

$$L/a > 1/\beta^2 \quad (39)$$

From (35) and (36) and assuming $\beta \approx \alpha\beta_0$ with $\alpha \approx$ 2–3, we find

$$\frac{E_z(z=0)}{E_a} \approx \frac{L_z\Delta\sigma_H}{c}$$

Taking $L_z \approx$ 3–5 km and $\Delta\sigma_H \approx 10^4$ s^{-1}, we find $[E_z(z = 0)]/E_a \approx$ .2–.5. Since $E_a \approx$ 10–20 mV m^{-1}, fields of a few millivolts per minute can be

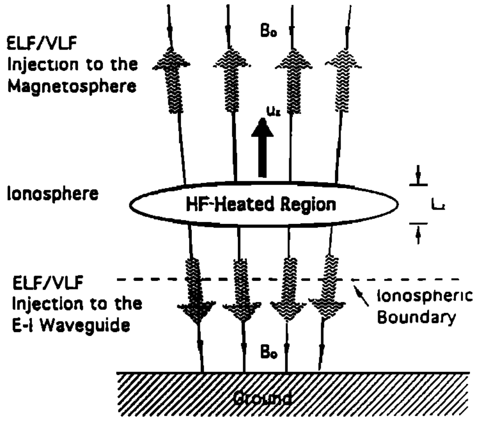

Figure 2. Cerenkov emission by a moving current source along the magnetic field in the ionosphere.

produced by Cerenkov emission. These far exceed the electric field values measured in experiments using amplitude modulation [*Stubbe et al.*, 1981]. Before closing we should remark on two issues.

First, we should distinguish the Cerenkov process discussed here from the one discussed by *Papadopoulos et al.* [1994b]. In the latter the excited modes were the eigenmodes of the ionospheric plasma, the whistler, and the helicon mode. The required speed of the current source was much smaller than the speed of light ($v/c \approx 0$ of the order of 10^{-2}). The excited waves propagate mainly upward (Figure 2), while the excitation of the EIW modes is indirect. The Cerenkov scheme discussed here leads to direct excitation of the waveguide modes and requires source motion at speeds comparable to the speed of light. Luminous as well as supraluminous speeds of the heater spot can be accomplished by appropriately phasing the heater array or operating in an interferometer mode [*Kotik et al.*, 1986]. The HAARP transmitter has a rapid scan mode of 10 μs and when operating on the high-gain mode can achieve effective sweeping speeds of 3–4 times the speed of light. In practice, speeds of the order of one half the speed of light will be required for excitation of the TM_0 mode.

Second, a proper analysis of the problem requires consideration of the ionospheric density and collision frequency profile. The equations describing the waveguide modes for exponential conductivity profiles including magnetization have been discussed by *Greigfinger and Greigfinger* [1979] and *Tripathi et al.* [1982]. We are currently using these techniques to examine the role of the inhomogeneities to the Cerenkov excitation. The analysis of the present paper should be considered a good analysis of the problem for sharp ionospheric density profiles, when the height at which the conduction current equals the displacement current is to within a skin depth of the reflection height. In concluding we must emphasize that by repetitive excitation of the pulses we can synthesize harmonic or other waveforms with the desirable low-frequency characteristics.

Appendix

We examine here the restrictions imposed in our analysis by the fact that we assumed a heated spot moving uniformly along the y axis from $y = -\infty$ to $y = +\infty$. To examine the effect of a finite sweeping distance Δy, we consider the following model. We assume that the heated spot is stationary for $t \le t_1 = -\Delta t/2$ (the coordinate of the spot is $y_1 = \Delta y/2$). During the time interval $-\Delta t/2 < t < \Delta t/2$ the spot moves uniformly and rectilinearly along the y axis with the speed v. From $t = t_2 = -\Delta t/2$ the spot is again stationary and its center is localized at $y_2 = \Delta y/2$. We perform the Fourier transformation using the above function $y(t)$. We obtain that in the case of finite distance the δ function in (12) should be replaced by δ_ω

$$\delta_\omega = \frac{1}{2\pi}\frac{\sin(\omega - k_y v)\Delta t/2}{\omega - k_y v} + \mu \qquad \text{(A1)}$$

where the function μ takes into account the contribution of the limits of integration $\pm\Delta t/2$ and does not correspond to the propagating wave. As a result, if $\sigma/\omega \gg 1$, we obtain

$$E_z(0) = (4\pi i/cz_0)E_a \int\int\int d\omega\, dk_x\, dk_y\, \Delta\Sigma\sqrt{-ik_y} \cdot \frac{\sin(\omega - k_y v)\frac{\Delta t}{2}}{2\pi(\omega - k_y v)} \frac{\sqrt{k_0^2\varepsilon - k_y^2}}{\varepsilon\omega} \frac{k_y}{k_0^2 - k_y^2} e^{-i(k_y y + k_x x - \omega t)} \qquad \text{(A2)}$$

We represent now the frequency of radiation ω in the form $\omega = kv + \Omega$, where Ω is a small correction due to the finite distance of motion. Substituting this expression in (41) of the main text, we find

$$E(0) = \frac{\sqrt{\pi} i E_a}{c^{3/2} z_0 \beta\sqrt{\sigma}} \int dk_y\, e^{-ik_y y} \Delta\Sigma\sqrt{-ik_y} \int d\Omega$$

$$\cdot \frac{\sin(\Omega\Delta t)}{\Omega} e^{i(k_y v+\Omega)t - i\sqrt{k_y^2\beta^2 + \frac{k_y v\Omega}{c^2}}} \quad (A3)$$

The obtained above result (A3) is valid if the corrections to $k_x = \beta k_y$ is small. It implies that the condition

$$\Omega \ll c k_y \beta^2 \quad (A4)$$

should be fulfilled. Taking for the estimations $\Omega \ll 2c/\Delta L$ and $k_y \sim 1/a$, we find that the condition for the validity of our results is given by

$$\Delta y \gg a/\beta^2 \quad (A5)$$

If the inequality (A4) is fulfilled, the phase front in (A2) is approximately a plane wave propagating at the angle $\alpha = (\pi/2) - \beta$ to the y axis. In the opposite case $\Delta y \ll a/\beta^2$ it is easy to verify that the Cerenkov radiation is absent. So in the optimum conditions ($\beta \ll 1$) the distance Δy of the heated spot motion should be considerably larger than the characteristic wavelength $\lambda = \pi a$.

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